GREYC
Electronics and Computer Science Laboratory

PhD thesis subject

Topological Analysis of 4D Spatio-Temporal Images by Morphological Hierarchies (Top4DSTeaM)

Hosting laboratory and research team: GREYC Laboratory - UMR 6072, IMAGE Team

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Context

Today, 3-dimensional (3D) image sequences are used in many scientific fields, including materials science, medicine and biology. A challenge with such spatio-temporal 3D images is to automatically detect the structural changes (of objects) in the 3D images over time. For instance, in materials science, geomecanicians are interested in studying kinematics of an assembly of particles for modeling complex material deformations, such as shear bands. The experimental approach combined with X-ray 3D computed tomography (CT) imaging allows them to acquire sequences of 3D scans of such material deformation (see Fig. 1 for a shear band case), where they study how contacting particle network evolves [1, 2].

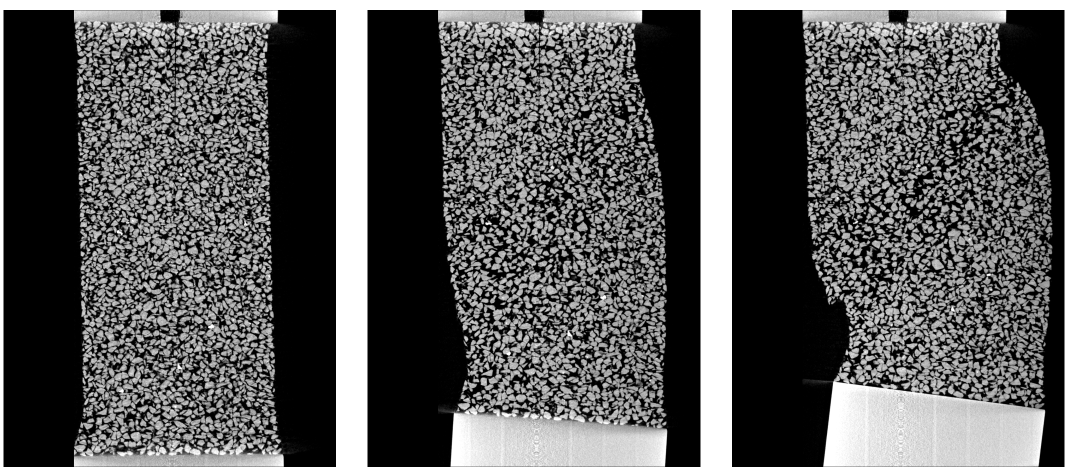

Figure 1: Cross-sections of 3D CT images at different times from a sequence of a deformed cylindrical sand sample, acquired during the triaxial compression test that induces a shear band.

Objectives

Among various types of changes occurring in 3D image sequences, we focus on the issues of topological changes, which underlie other geometric information. These topological issues in image analysis are crucial: understanding the topology of objects in images and their evolution over time is a desirable property in many image processing applications. These topological questions have received little attention to date. We aim to quantify topological properties and to propose new methods that best detect such topological changes.

To our knowledge, 4-dimensional (4D, i.e. 3D+time) images are rarely considered. In particular, it is difficult to use deep learning paradigms due to the memory requirements induced by the huge size of the data and the paucity of available (annotated) 4D datasets. Recently, pioneering work has been done on how to generate data for various simple topological shapes and how to learn 4D topological features using them [3]. Certainly, the generated data are far from being real images such as those shown in Fig. 1. Besides, 4D geometric and topological information is not usually taken into account in analysis of 3D image sequences; indeed, the time dimension is generally ignored, or only two consecutive images are considered. However, the benefit of taking 4D topology into account has been demonstrated, for example, in segmentation of 4D cardiac images [4]; a 3D object that deforms over time can be seen as a 4D (3D+time) object, which leads to a topological constraint in 4D. In the same way, rigid 3D particles that move over time, as in Fig. 1, can be seen as tubes in 4D. Then, the evolution of the network of 3D particles in contact can be interpreted as the way in which these 4D tubes intertwine. Our question will therefore be how to measure the topological complexity of these 4D interlaced tubes.

These issues, considered here in the applicative context of materials science, also occur in other context. This is especially the case in biological imaging, for instance in the quantitative assessment of living cells evolution in 2D images [5] or in modeling the evolution of vascular microstructures in very high resolution 3D synchrotron images [6] where one of the dimensions can be seen a pseudo-time dimension due to strong continuity.

State of the art

Topology is the branch of mathematics that studies the properties of geometrical objects preserved under continuous deformation without tearing or gluing. Various topological descriptors and invariants for objects exist, among which Betti numbers b_k (for nD, the number of connected components b_0 and the number of kD holes b_k, $k = 1, \ldots, n-1$) are simple ones. The frameworks of digital topology [7] and of cubical complexes [8] allow us to compute those Betti numbers for binary images. With respect to gray images, persistent homology [9], which has been recently popularized and extensively used in various fields to study the shape and structure of data, allows us to define persistent Betti numbers for gray images [10], but also to observe the evolution of the homology groups over the grey-levels. However, these numerical or vectorial descriptors do not offer enough information for detecting topological changes in an image sequence. If some Betti numbers / homology groups vary along the sequence, this ensures that there exist some topological changes in the sequence while this does not tell where we can find them in the images. Besides, it is also well known that maintaining these numerical or vectorial descriptors in a discrete sequence does not even guarantee topological equivalence. This information loss is an obstacle to analyzing spatio-temporal images.

In order to overcome this shortcoming, tree-structural topological descriptors/invariants based on mathematical morphology [11], which are adapted for modeling not only binary but also gray images in higher dimensions, are commonly used for characterizing topological changes:

- adjacency tree for binary images [12];
- component tree (max-tree and its dual, min-tree) for gray images [11], also called merge trees [13] in topological data analysis;
- tree of shapes [14], which is a tree of object boundaries in an image, seen as a hierarchical structure containing the information of both max- and min-trees, also called contour tree [15] in topological data analysis.

In this line, the topological tree of shapes, which is based on the tree of shapes, married with digital topological concept with the aim of topological simplification, has been introduced recently [16], with reasonable computational time for its construction [17] via efficient construction of tree of shapes [18]. Based on these properties, it may constitute a relevant starting point of this thesis.

In order to quantify topological differences by using above hierarchical representations, distance between trees are necessary. Various such distances already exist. One can cite, non-exhaustively, tree edit distances [19], interleaving distance between merge trees [20], Gromov-Haussdorff distance between metric trees [21], Wasserstein distance between merge trees [22], optimal transport distance on a tree metric [23], and their variations. However, those distances do not take into account image-related information carried by each node (for example, region similarity based on region size, shape, position, intensity, etc.) even though the structure of tree of shapes is richer than a simple tree. In this context, a new distance between trees of shapes by measuring image-related similarity between nodes using the Hausdorff distance has been proposed [24]. Similarity measures should depend on applications we consider.

Issues and challenges

The objective of this thesis will be to develop methods and tools to characterize and quantify the topological changes of 4D spatio-temporal grayscale images. As we note that the initial work has been made, such as topological tree of shapes [16], we will first explore its capacity for topological measures, inspired by the distance based on the node matching from Hausdorff distance [24]. We aim at compensating the weaknesses inherent to the measure by revisiting some approaches of mathematical morphology [11], digital topology (for grayscale images in particular) [25] and topological data analysis [9].

Beforehand, the first issue we need to address is how to simplify the generated tree as a topological description of a given image (sequence). In fact, it is observed that a generated tree generally contains many nodes, when each pixel/voxel has a different value from its neighbors. This can be seen as "topological noise", so we should remove insignificant nodes to simplify the tree, so that it contains only meaningful information. In other words, we need to define what "topological noise" is to proceed topological denoising. Removing nodes of component trees is easy (elements of a removing node joining its parent), so that various region-based morphological filtering techniques, called connected operators, exist [26]. They have good contour-preserving properties such that they cannot create new contours nor modify their position. However, removing nodes of tree of shapes is not as simple as component trees; a solution for this problem has been proposed recently [27] as a sequence of elementary operations. This idea may help us to ensure correct topological denoising for 4D spatio-temporal images.

The second issue is how to treat tunnels (kD holes where $k = 1$ for 3D and $k = 1, 2$ for 4D) with tree-structural topological descriptors. In the case of 3D binary images, the idea of adding the tunnel information as the weight of an edge of the adjacency tree has been introduced [28]. It is a very ambitious challenge if we can extend this idea to 4D grayscale images.

Course of the thesis

We will first explore the problem in three dimensions (or "2D+t") and then in four dimensions ("3D+t"), which is an ambitious challenge as many problems still remain open in this framework. The thesis consists of the following three steps:

1. **Theoretical aspects:** as mentioned above, we choose the topological tree of shapes [16] as a topological descriptor which is a relevant starting point. We will explore its capacity for topological measures, inspired by the distance based on the node matching based on Hausdorff distance [24]. Topological image denoising based on this tree, which generally corrupts the tree structure, is also necessary prepossessing for its practical use. Equipped with these tools for quantifying topological changes, we will then focus on the development of topological change detection methods, with minimizing "topological noise".

2. **Computational aspects:** analyze and improve the computational efficiency of our method for topological change detection from a 4D image using the hierarchical topological descriptor, and also to measure the topological difference between two images. One of the key issues will be how to use the tree structure for efficient computation. On a technical level, there are already several computational tools for each domain, for example:

   - for discrete topology: DGtal, Pink
   - for topological data analysis: Gudhi, TTK
   - for mathematical morphology: HIGRA

   A new tool (or module) will be developed (or added) by relying on these tools.

3. **Application aspects:** the proposed concepts will be validated in the applicative context of geomecanics in collaboration with Gioacchino Viggiani (Laboratory 3SR, Grenoble) and Edward Ando (Center of Imaging, EPFL) who provide 3D CT image sequences, as shown in Fig. 1. Their interest is to characterize the evolution of contacting particle network, i.e. kinematics of particles. The applicative challenge of this thesis is to obtain related topological and geometrical informations [29] from the 4D spatio-temporal images efficiently and precisely by using morphological hierarchies based on the above theoretical and computational studies.

Expected results

Topological data analysis is now popular and used in various fields of data science. The subject of this thesis is very ambitious, since we aim to deal with 4D images, which are very large and rarely treated so far. The thesis also includes theoretical and computational aspects with attractive applications with real data, which provide many interesting open questions.

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Profile
We are looking for a highly motivated candidate who holds (will hold) a diploma of Master/engineering in computer science or applied mathematics. Candidates should be comfortable with programming in C++ and python.

Application
To apply, email the supervisors a dossier containing a CV, covering letter, transcripts of the last two years of study, and possibly letters of recommendation or reference names.

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