GREYC
Laboratoire de recherche en sciences du numérique

Master 2 Internship subject - 2025

# Topological Analysis of 4D Spatial-Temporal Images by Morphological Hierarchies

**Hosting laboratory and research team:** GREYC Laboratory - UMR 6072, IMAGE Team

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**Period :** 5-6 months (and possible thesis continuation)

**Keywords :** Topological analysis, 4D images, Digital topology, Mathematical morphology, Persistent homology

## Context and issues

Today, 3-dimensional (3D) image sequences are used in many scientific fields, including materials science, medicine and biology. A challenge with such spatial-temporal 3D images is to automatically detect the structural changes (of objects) in the 3D images over time. For instance, in materials science, geo-mechanists are interested in studying kinematics of an assembly of sand grains for modeling complex material deformations, such as shear bands. The experimental approach combined with X-ray 3D computed tomography (CT) imaging allows them to acquire sequences of 3D scans of such material deformation (see Fig. 1 for a shear band case), where they study how contacting grain network evolves [1, 2].

Among various types of changes occurring in 3D image sequences, we focus on the issues of topological changes, which underlie other geometric information. These topological issues in image analysis are crucial : understanding the topology of objects in images and their evolution over time is a desirable property in many image processing applications. These topological questions have received little attention to date. The aim of this internship is to quantify topological properties and to propose new methods that best detect such topological changes.

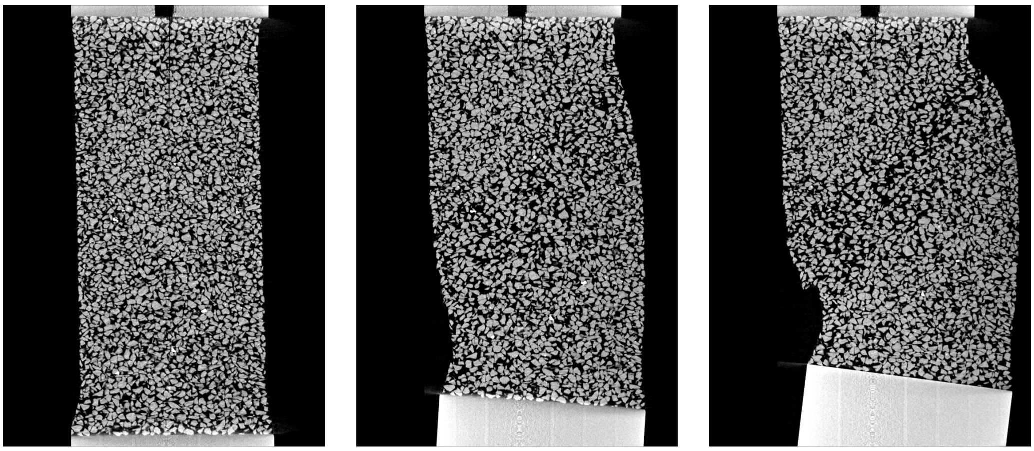

FIGURE 1 – Cross-sections of 3D CT images at different times from a sequence of a deformed cylindrical sand sample, acquired during the triaxial compression test that induces a shear band.

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**State of the art**

To our knowledge, 4-dimensional (4D, i.e. 3D+time) images are rarely considered. In particular, it is difficult to use deep learning paradigms due to the memory requirements induced by the huge size of the data and the paucity of available (annotated) 4D datasets. Besides, 4D geometric and topological information is not usually taken into account in analysis of 3D image sequences ; indeed, the time dimension is generally ignored, or only two consecutive images are considered. However, taking 4D topology into account has the following advantage ; rigid 3D particles that move over time can be seen as tubes in 4D. Then, the evolution of the network of 3D particles in contact can be interpreted as the way in which these 4D tubes intertwine. Our question will therefore be how to measure the topological complexity of these 4D interlaced tubes.

Among various topological descriptors and invariants for objects in gray images, persistent homology, which has been recently popularized and extensively used in various fields to study the shape and structure of data, allows us to define persistent Betti numbers, but also to observe the evolution of the homology groups over the grey-levels [3]. However, these numerical or vectorial descriptors do not offer more complex information for detecting topological changes in an image sequence. In order to overcome this shortcoming, tree-structural topological descriptors are commonly used for characterizing topological changes for grayscale images [4]. In this line, the topological tree of shapes [5], which is recently proposed for topological simplification of grayscale images based on mathematical morphology, accompanied with reasonable computational time for its construction [6], may constitute a relevant starting point of this internship.

**Objectives**

The aim of this internship will be to develop tools to characterize and quantify topological changes in 4D spatio-temporal images in the context of the application of geo-mecanics as seen in Fig. 1. More concretely, our aim is not only to evaluate the movement of individual grains, but also to analyze the contacts between them. In particular, we are interested in how grain contacts evolve over time as a result of grain movements. To this end, we will use topological tree of shapes [5], recently proposed topological descriptor based on morphological hierarchies. To begin with, we build a topological tree of shapes for each 3D image, and compare the trees to identify their differences. We will then study the temporal tracking of the topological structures with additional geometric and topological constraints.

The integration of developments in the Higra (Hierachical Graph Analysis) library [7] is envisaged.

**Profile**

We are looking for a highly motivated student of Master 2/last year of engineering school in computer science or applied mathematics. Candidates should be comfortable with programming in C++ and python.

**Application**

To apply, email the supervisors a dossier containing a CV, covering letter, transcripts of the last two years of study, and possibly letters of recommendation or reference names.

**References**


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